Technical Notes

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Dimensionless Governing Equations for Vapor and Liquid Flow Analysis of Heat Pipes

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Nomenclature

C = specific heat, J/kg K
C_E = Ergun's coefficient
c = heat capacity, ρC, J/m^3 K
Da = Darcy number, K/L_h^2
D_H = hydraulic mean diameter, m
d = pore diameter, m
Ec = Eckert number, $U^2/C(T_w - T_0)$
Fr = Froude number, U^2/gL_h
g = gravitational constant, m/s^2
h = convective heat-transfer coefficient, W/m^2 K
h_{fg} = latent heat of vaporization, J/kg
K = permeability of the porous medium, m^2
k = thermal conductivity, W/mK
L = length, m
Pe = Peclet number, $= RePr$
Pr = Prandtl number, $C\mu/k$
p = pressure, N/m^2
Re = Reynolds number, $\rho u D_H/\mu$
Re_K = Reynolds number based on permeability of the porous medium, $\rho u\sqrt{K}/\mu$
$\hat{R}e_K$ = Reynolds number based on the pore permeability and interstitial fluid velocity, $\rho u\sqrt{(K/\varphi)}/\mu$
Sr = Strouhal number, L_h/Ut_0
T = temperature, K
t = time, s
U = characteristic velocity, m/s (u_0 or u_k defined in the text)
u = x-velocity component, m/s
V = resultant velocity, m/s
v = y-velocity component, m/s
w = z-velocity component
x = axial direction, axial distance, m
y = transverse (depth) direction, transverse distance, m
z = width direction, width, m
α = thermal diffusivity, m^2/s
Γ = length to hydraulic diameter ratio, L_h/D_H
δ = thickness, m
θ = dimensionless temperature
λ = second coefficient of viscosity, N s/m^2
μ = dynamic viscosity, N s/m^2
ν = kinematic viscosity, m^2/s
ρ = density, kg/m^3
Φ = dissipation function, 1/s^2
φ = porosity of the wick material
∞ = freestream

Subscripts

eff = effective
f = fluid
h = heat pipe
K = based on permeability
l = liquid
m = mean
o = initial (reference) value
p = porous medium
s = solid
v = vapor
w = wall

Superscript

$*$ = dimensionless value

Introduction

THE energy transport and momentum equations in dimensional form impose considerable restrictions when applied to the computational analysis of heat pipes and other similar capillary pumped devices. A review of the literature to date indicates that only a very few investigations have attempted to develop an appropriate nondimensional form of the governing equations, applicable to the vapor core and wick analysis of these devices.[1,2] The current work presents in general dimensionless form the governing equations for heat-pipe analysis, including the solid wall, the wick, and the vapor core. This general formulation can be applied for laminar steady-state and unsteady-state processes and is applicable to both incompressible and compressible vapor flow in heat pipes.

Unlike the case of fully developed convective heat transfer, the analysis of heat pipes presents no fixed velocity in the domain that is known a priori, or that can be computed directly. This, when coupled with the difficulties involved in the identification of the characteristic dimensions, especially for the fluid velocities, makes it difficult to apply nondimensional analyses to these problems. Moreover, given that most heat pipes have asymmetric heating, the possibility of axial conduction in the heat-pipe wall and a relatively complicated physical structure, make determination of the location of the maximum velocities difficult until the computational analysis has been completed.

The governing differential equations for the flow and heat transfer in the heat pipe are the continuity, momentum, and energy equations

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in the vapor core and the wick regions. In a saturated wick, the momentum equations are extended to include the Darcy, Forchheimer and Brinkman terms, and the void fraction.[3] In these applications, the wick plays an important role in the performance and a wide variety of porous media have been used as the wick material. The range of pore sizes can be of the order of molecular scales to millimeters or larger for artery wicks. Four important length scales are useful in the analysis of heat transfer in the saturated porous medium. These are the pore size d, the linear dimension of the representative volume l, the system dimension L (L_h in the present case), and the Brinkman screening distance $K^{1/2}$. When the ratio $L/d \approx 0$, no assumption can be made about the existence of a local thermal equilibrium between the finite volumes of the phases; when $L/d \gg 1$ and when the variation of temperature across d is negligible compared to that across L, we can assume that within a distance d both phases are in thermal equilibrium.[3] The $K^{1/2}$ scale length mentioned before, according to Nield and Bejan,[4] represents the effective pore diameter, which could be used in the definition of the Reynolds number in the porous medium. Kaviany[3] pointed out that $K^{1/2}$ is smaller than the porous diameter d, with an approximate order of magnitude $\mathcal{O}(10^{-2}\,d)$. The four length scales are in the following order of magnitude (with a loose requirement for the presence of local thermal equilibrium) $K^{1/2} \ll d \ll l \ll L$.

The mean heat capacity and the effective thermal conductivity of the porous media can be calculated based on appropriate models. The Darcy number appears in the dimensionless equations to account for the relative magnitudes of the characteristic lengths, namely, $K^{1/2}$ and L_h. This is because the Reynolds number is defined based on $K^{1/2}$, and the dimensionless distances are based on the heat pipe length L_h. Similarly, in the formulation for the governing equations for the vapor core, a length parameter $\Gamma = L_h/D_H$ appears, as the Reynolds number is defined based on the hydraulic mean diameter.

Fand et al.[5] identified four different flow regimes of flow through porous media: pre-Darcy, Darcy, Forchherimer, and turbulent. Kececioglu et al.[6] provided experimental evidence for determining the demarcation criteria in the flow of water through a bed of randomly packed spherical beads. They found that the pre-Darcy flow occurred for $\hat{R}e_K < 0.06$, Darcy flow for $0.06 < \hat{R}e_K < 0.12$, post-Darcy laminar Forchheirmer flow for $0.34 < \hat{R}e_K < 2.30$, and turbulent flow for $\hat{R}e_K > 3.4$. Studies of the flow regimes in porous media, based on the pore Reynolds number, have been reported in the literature.[7,8]

The reference velocity in the porous medium u_K will be based on the limitation on the flow regime in the porous medium. Founded on the results given by Kececioglu et al.,[6] it is possible to select a characteristic Reynolds number for a particular flow regime. Similarly, the reference velocity in the vapor core u_0 can be obtained based on the laminar flow regime in the vapor core.

A useful method for obtaining dimensionless equations might be to recast the governing equations with characteristic velocities derived, based on the maximum Reynolds number, with definitions appropriate to the vapor core and the wick, which could be fixed in a problem, based on the flow regime. Where the Reynolds number appears as a parameter in the derived nondimensional equations, the values of the Reynolds number could be used as an input in the solution. This would restrict the vapor flow to the laminar regime, which is the focus of this analysis. Because the flow is induced by capillary pumping, the liquid flow regime in the wick can be assumed to always be laminar,[9] with a Reynolds number less than 2.3×10^3 and a Mach number less than 0.2. The vapor flow, however, can be either laminar or turbulent. For laminar flow in the vapor core, the compressibility effects can typically be neglected. For a Mach number greater than 0.2, the compressibility effect must be considered in the estimation of the pressure drop in the vapor core; some correlations for this situation can be found in Ref. 9.

The importance of the representative dimensionless parameters in the governing equations for heat-pipe analysis presented here is explained as follows: The Froude number, which includes the gravity, is an important parameter because in a heat pipe a free surface is present between the porous medium and the vapor core. The Strouhal number plays an important role while dealing with transient processes.[10] The Eckert number is closely related to the Mach number; thus, for ideal gases $Ec = (1-\gamma)Ma^2 T_\infty/(T_w - T_\infty)$. This determines the presence of compressibility effects; if the Mach number is small compared to unity, the vapor flow can be considered as incompressible.[11,12] The sonic limit in a heat pipe can be estimated by analyzing the effects of this parameter. The Peclet number in a heat pipe can vary depending on the working fluid. This parameter is associated with the growth of the thermal boundary layer, and, in general, the Nusselt number can be expressed in terms of the Peclet number, as in forced convection heat transfer.[13]

Methodology

Characteristic Dimensions

The characteristic dimensions used in the procedure of nondimensionalizing the governing equations are given next:

Density:

$$\rho^* = \rho/\rho_0$$

Heat capacity:

$$c^* = c/c_0, \qquad c_0 = \rho_0 C_0$$

Velocities in the vapor core:

$$u_v^* = u_v/u_0, \qquad v_v^* = v_v/u_0, \qquad w_v^* = w_v/u_0$$

$$u_0 = Re_v \mu_v/\rho_v D_H$$

Velocities in the wick:

$$u_l^* = u_l/u_K, \qquad v_l^* = v_l/u_K, \qquad w_l^* = w_l/u_K$$

$$u_K = \hat{R}e_K \mu_l \Big/ \rho_l (K/\varphi)^{\frac{1}{2}}$$

For laminar flow, the limiting values of the Reynolds numbers in the vapor core and the wick can be used in the preceding expressions.

The resultant velocities in the vapor core and the wick are

$$V_v^* = V_v/u_0, \qquad V_v = \left(u_v^2 + v_v^2 + w_v^2\right)^{0.5}$$

$$V_l^* = V_l/u_K, \qquad V_l = \left(u_l^2 + v_l^2 + w_l^2\right)^{0.5}$$

Time:

$$t^* = t/t_0, \qquad t_0 = L_h/u_0$$

Length dimensions:

$$x^* = x/L_h, \qquad y^* = y/L_h, \qquad z^* = z/L_h$$

The dimensionless temperature profile is defined as follows:

$$\theta = (T - T_0)/(Tw - T_0)$$

The mean heat capacity in the wick (fluid saturated porous medium) is calculated under the assumption of local thermal equilibrium between the solid (porous media) and fluid phases[4]:

$$(\rho c)_m = (1-\varphi)\rho_s c_s + \varphi \rho_f c_f$$

For a Newtonian fluid the dissipation function is defined as

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2$$

Table 1 Governing equations in the dimensional and dimensionless forms

Description	Governing equations
Case 1: General compressible flow formulation, assuming ideal gas. (For vapor core: dimensional form)	Vapor core: dimensional form
	Continuity:
	$\frac{\partial \rho_v}{\partial t} + \nabla \cdot (\rho_v \bar{\boldsymbol{V}}) = 0$ (1)
	Momentum:
	$\frac{\partial (\rho_v \bar{\boldsymbol{V}})}{\partial t} + \nabla \cdot (\rho_v \bar{\boldsymbol{V}} \bar{\boldsymbol{V}}) = \rho_v \boldsymbol{g} - \nabla p_v + \mu_v \nabla^2 \bar{\boldsymbol{V}} + (\mu_v + \lambda_v) \nabla (\nabla \cdot \bar{\boldsymbol{V}})$ (2)
	Energy:
	$\frac{\partial}{\partial t}(c_v T) + \nabla \cdot (c_v \boldsymbol{V} T) = \nabla \cdot (k_v \nabla T) - p_v (\nabla \cdot \boldsymbol{V}) + \mu_v \Phi_v + \rho_v T \frac{DC_v}{Dt}$ (3)
	Vapor core: dimensionless form
	Continuity:
	$Sr_v \frac{\partial \rho_v^*}{\partial t^*} + \nabla^* \cdot \left(\rho^* \bar{\boldsymbol{V}}_v^*\right) = 0$ (4)
	Momentum:
	$Sr_v \frac{\partial \left(\rho_v^* \bar{\boldsymbol{V}}^*\right)}{\partial t^*} + \nabla^* \cdot \left(\rho_v^* \bar{\boldsymbol{V}}^* \bar{\boldsymbol{V}}^*\right) = \frac{1}{Fr_v} \rho_v^* \boldsymbol{g}^* - \nabla^* p_v^* + \frac{\rho_v^*}{\Gamma Re_v} \nabla^{*2} \bar{\boldsymbol{V}}^* + \frac{\rho_v^* \left(\mu_v^* + \lambda_v^*\right)}{3 \Gamma Re_v} \nabla^* (\nabla^* \cdot \bar{\boldsymbol{V}}^*)$ (5)
	Energy:
	$\frac{\partial}{\partial t^*}\left(c_v^* T^*\right) + \nabla^* \cdot \left(c_v^* \boldsymbol{V}^* T^*\right) = \frac{1}{Pe_v \Gamma} \nabla^* \cdot \left(k_v^* \nabla^* T^*\right) - Ec_v p^* (\nabla^* \cdot \boldsymbol{V}^*) + \frac{Ec_v}{Re_v \Gamma} \Phi_v^* + \rho_v^* T^* \frac{DC_v^*}{Dt^*}$ (6)
Case 1: General compressible flow formulation, assuming Newtonian fluid. (For porous wick: dimensional form)	Porous wick: dimensional form
	Continuity:
	$\varphi \frac{\partial \rho_v}{\partial t} + \nabla \cdot (\rho \boldsymbol{V}_v) = 0$ (7)
	Momentum:
	$\varphi^{-1} \left[\frac{\partial (\rho_v \boldsymbol{V}_v)}{\partial t} + \nabla \cdot (\rho_v \boldsymbol{V}_v \boldsymbol{V}_v)\right] = \rho_v \boldsymbol{g} - \nabla p_v + \frac{\mu_v}{\varphi} \nabla^2 \boldsymbol{V}_v - \frac{\mu_v}{K} \boldsymbol{V}_v - \frac{C_E \rho_v}{K^{0.5}} \lvert \boldsymbol{V}_v \rvert \boldsymbol{V}_v$ (8)
	Energy:
	$\frac{\partial}{\partial t}(c_m T) + \nabla \cdot (c_v T \bar{\boldsymbol{V}}) = \frac{DP}{Dt} + \nabla \cdot (k_{\text{eff}} \nabla T) + \rho_v T \frac{DC_v}{Dt} + \varphi \mu_v \Phi_v$ (9)
	Porous wick: dimensionless form
	Continuity:
	$Sr_v \varphi \frac{\partial \rho_v^*}{\partial t^*} + \nabla^* \cdot \left(\rho_v^* \bar{\boldsymbol{V}}_v^*\right) = 0$ (10)
	Momentum:
	$\frac{1}{\varphi}\left[Sr_v \frac{\partial \left(\rho_v^* \bar{\boldsymbol{V}}_v^*\right)}{\partial t^*} + \nabla^* \cdot \left(\rho_v^* \bar{\boldsymbol{V}}_v^* \bar{\boldsymbol{V}}_v^*\right)\right] = \frac{1}{Fr_v} \rho_v^* \boldsymbol{g} - \nabla^* p_l^* + \frac{Da^{\frac{1}{2}} \rho_v^*}{\varphi Re_K} \nabla^{*2} \bar{\boldsymbol{V}}_v^* - \frac{\rho_v^*}{Re_K Da^{\frac{1}{2}}} \bar{\boldsymbol{V}}_v^* - \frac{C_E \rho_v^*}{Da^{\frac{1}{2}}} \left\lvert \bar{\boldsymbol{V}}_v^* \right\rvert \bar{\boldsymbol{V}}_v^*$ (11)
	Energy:
	$\frac{\partial}{\partial t^*}\left(c_m^* T^*\right) + \nabla^* \cdot \left(c_v^* T^* \bar{\boldsymbol{V}}^*\right) = Ec_K \frac{DP^*}{Dt^*} + \frac{Da^{\frac{1}{2}}}{Pe_K} \nabla^* \cdot \left(k_{\text{eff}}^* \nabla^* T^*\right) + \rho_v^* T^* \frac{DC_v^*}{Dt^*} + \varphi \frac{Ec_K Da^{\frac{1}{2}}}{Re_K} \Phi_v^*$ (12)
Case 2: Conservative form for incompressible flow	Vapor core: dimensional form
	Continuity
	$\frac{\partial \rho_v}{\partial t} + (\bar{\boldsymbol{V}} \cdot \nabla) \rho_v = 0$ (13)
	Momentum:
	$\frac{\partial (\rho_v \bar{\boldsymbol{V}}_v)}{\partial t} + \nabla \cdot (\rho_v \bar{\boldsymbol{V}}_v \bar{\boldsymbol{V}}_v) = \rho_v \boldsymbol{g} - \nabla p_v + \mu_v \nabla^2 \bar{\boldsymbol{V}}_v$ (14)
	Energy:
	$\frac{\partial}{\partial t}(c_v T) + \nabla \cdot (c_v \boldsymbol{V} T) = \nabla \cdot (k_v \nabla T) + \mu_v \Phi_v + \rho_v T \frac{DC_v}{Dt}$ (15)
	Vapor core: dimensionless form
	Continuity:
	$Sr_v \frac{\partial \rho_v^*}{\partial t^*} + (\bar{\boldsymbol{V}}^* \cdot \nabla^*) \rho_v^* = 0$ (16)
	Momentum:
	$Sr_v \frac{\partial \left(\rho_v^* \bar{\boldsymbol{V}}_v^*\right)}{\partial t^*} + \nabla^* \cdot \left(\rho_v^* \bar{\boldsymbol{V}}_v^* \bar{\boldsymbol{V}}_v^*\right) = \frac{1}{Fr_v} \rho_v^* \boldsymbol{g} - \nabla^* p_v^* + \frac{\rho_v^*}{\Gamma Re_v} \nabla^{*2} \bar{\boldsymbol{V}}_v^*$ (17)
	Energy:
	$\frac{\partial}{\partial t^*}\left(c_v^* T^*\right) + \nabla^* \cdot \left(c_v^* \boldsymbol{V}^* T^*\right) = \frac{1}{Pe_v \Gamma} \nabla^* \cdot \left(k_v^* \nabla^* T^*\right) + \frac{Ec_v}{Re_v \Gamma} \Phi_v^* + \rho_v^* T^* \frac{DC_v^*}{Dt^*}$ (18)

(Continued)

Table 1 Governing equations in the dimensional and dimensionless forms (continued)

Description	Governing equations	
Case 2: Conservative form for incompressible flow	Porous wick: dimensional form	
	Continuity:	
	$\varphi \frac{\partial \rho_l}{\partial t} + (\boldsymbol{V}_l \cdot \nabla)\rho = 0$	(19)
	Momentum:	
	$\varphi^{-1}\left[\frac{\partial(\rho_l \boldsymbol{V}_l)}{\partial t} + \nabla \cdot (\rho_l \boldsymbol{V}_l \boldsymbol{V}_l)\right] = \rho_l \boldsymbol{g} - \nabla p_l + \frac{\mu_l}{\varphi}\nabla^2 \boldsymbol{V}_l - \frac{\mu_l}{K}\boldsymbol{V}_l - \frac{C_E \rho_l}{K^{0.5}}\lvert\boldsymbol{V}_l\rvert\boldsymbol{V}_l$	(20)
	Energy:	
	$\frac{\partial}{\partial t}(c_{\text{eff}}T) + \nabla \cdot (c_l T \bar{\boldsymbol{V}}) = \nabla \cdot (k_{\text{eff}} \nabla T) + \rho_l T \frac{DC_l}{Dt} + \varphi \mu_l \Phi_l$	(21)
	Porous wick: dimensionless form	
	Continuity:	
	$Sr_l \varphi \frac{\partial \rho_l^*}{\partial t^*} + \left(\bar{\boldsymbol{V}}_l^* \cdot \nabla^*\right)\rho_l^* = 0$	(22)
	Momentum:	
	$\frac{1}{\varphi}\left[Sr_l \frac{\partial\left(\rho_l^* \bar{\boldsymbol{V}}_l^*\right)}{\partial t^*} + \nabla^* \cdot \left(\rho_l^* \bar{\boldsymbol{V}}_l^* \bar{\boldsymbol{V}}_l^*\right)\right] = \frac{1}{Fr_l}\rho_l^* \boldsymbol{g}^* - \nabla^* p_l^* + \frac{Da^{\frac{1}{2}} \rho_l^*}{\varphi Re_K}\nabla^{*2} \bar{\boldsymbol{V}}_l^* - \frac{\rho_l^*}{Re_K Da^{\frac{1}{2}}}\bar{\boldsymbol{V}}_l^* - \frac{C_E \rho_l^*}{Da^{\frac{1}{2}}}\left\lvert\bar{\boldsymbol{V}}_l^*\right\rvert\bar{\boldsymbol{V}}_l^*$	(23)
	Energy:	
	$\frac{\partial}{\partial t^*}\left(c_m^* T^*\right) + \nabla^* \cdot \left(c_l^* T^* \bar{\boldsymbol{V}}^*\right) = \frac{Da^{\frac{1}{2}}}{Pe_K}\nabla^* \cdot \left(k_{\text{eff}}^* \nabla^* T^*\right) + \rho_l^* T^* \frac{DC_l^*}{Dt^*} + \varphi \frac{Ec_K Da^{\frac{1}{2}}}{Re_K}\Phi_l^*$	(24)
Case 3: Nonconservative form applicable to incompressible flow	Vapor core: dimensional form	
	Continuity:	
	$\nabla \cdot \boldsymbol{V}_v^* = 0$	(25)
	Momentum:	
	$\frac{\partial \boldsymbol{V}_l}{\partial t} + \boldsymbol{V}_l \cdot (\nabla \boldsymbol{V}_l) = \boldsymbol{g} - \frac{1}{\rho_v}\nabla p_v + \nu_v \nabla^2 \boldsymbol{V}_l$	(26)
	Energy:	
	$c_v\left[\frac{\partial}{\partial t}(T) + \bar{\boldsymbol{V}} \cdot (\nabla T)\right] = \nabla \cdot (k_v \nabla T) + \mu_v \Phi_v$	(27)
	Vapor core: dimensionless form	
	Continuity:	
	$\nabla^* \cdot \boldsymbol{V}_v^* = 0$	(28)
	Momentum:	
	$Sr_v \frac{\partial \boldsymbol{V}_v^*}{\partial t^*} + \boldsymbol{V}_v^* \cdot \left(\nabla \boldsymbol{V}_v^*\right) = \frac{1}{Fr_v}\boldsymbol{g}^* - \frac{1}{\rho_v^*}\nabla p_v^* + \frac{1}{\Gamma Re_v}\nabla^{*2}\boldsymbol{V}_v^*$	(29)
	Energy:	
	$c_v^*\left[\frac{\partial}{\partial t^*}(T^*) + \nabla^* \cdot (\boldsymbol{V}^* T^*)\right] = \frac{1}{Pe_v \Gamma}\nabla^* \cdot \left(k_v^* \nabla^* T^*\right) + \frac{Ec_v}{Re_v \Gamma}\Phi_v^*$	(30)
Case 3: Nonconservative form applicable to incompressible flow	Porous wick: dimensional form	
	Continuity:	
	$\nabla \cdot \boldsymbol{V}_l = 0$	(31)
	Momentum:	
	$\varphi^{-1}\left[\frac{\partial \boldsymbol{V}_l}{\partial t} + \boldsymbol{V}_l \cdot (\nabla \boldsymbol{V}_l)\right] = \boldsymbol{g} - \frac{1}{\rho_l}\nabla p_l + \frac{\nu_l}{\varphi}\nabla^2 \boldsymbol{V}_l - \frac{\nu_l}{K}\boldsymbol{V}_l - \frac{C_E}{K^{0.5}}\lvert\boldsymbol{V}_l\rvert\boldsymbol{V}_l$	(32)
	Energy:	
	$c_{\text{eff}}\frac{\partial}{\partial t}(T) + c_l \boldsymbol{V} \cdot \nabla T = \nabla \cdot (k_{\text{eff}} \nabla T) + \varphi \mu_l \Phi_l$	(33)
	Porous wick: dimensionless form	
	Continuity:	
	$\nabla^* \cdot \boldsymbol{V}_l^* = 0$	(34)
	Momentum:	
	$\frac{1}{\varphi}\left[Sr_l \frac{\partial \boldsymbol{V}_l^*}{\partial t^*} + \boldsymbol{V}_l^* \cdot \left(\nabla \boldsymbol{V}_l^*\right)\right] = \frac{1}{Fr_l}g_z^* - \frac{1}{\rho_l^*}\nabla p_l^* + \frac{Da^{\frac{1}{2}}}{\varphi Re_K}\nabla^{*2}\boldsymbol{V}_l^* - \frac{1}{Re_K Da^{\frac{1}{2}}}\boldsymbol{V}_l^* - \frac{C_E}{Da^{\frac{1}{2}}}\left\lvert\boldsymbol{V}_l^*\right\rvert\boldsymbol{V}_l^*$	(35)
	Energy:	
	$c_m^* \frac{\partial}{\partial t^*}(T^*) + \left(c_l^* \boldsymbol{V}^*\right) \cdot \nabla^* T^* = \frac{Da^{\frac{1}{2}}}{Pe_K}\nabla^* \cdot \left(k_{\text{eff}}^* \nabla^* T^*\right) + \varphi \frac{Ec_K Da^{\frac{1}{2}}}{Re_K}\Phi_l^*$	(36)

Dimensionless Formulation

Case 1: Compressible Flow Formulation for Heat Pipes

The possibility of high Mach numbers can exist for heat pipes, particularly for liquid-metal heat pipes; thus, any general formulation of the problem should be based on the compressible flow equations in the conservative form.[14] The dimensionless form of the compressible flow equations of continuity, momentum, and energy are derived using the characteristic dimensions just discussed. The extended momentum equations for the porous medium[3,4] are utilized for modeling the heat-pipe wick. The set of nondimensional equations is shown as case 1 in Table 1. The equations for compressible flow in the wick are presented only for completeness; however, for liquid flow in the wick the incompressible flow formulation will be sufficient.

Case 2: Conservative Form of the Governing Equations for Incompressible Flow

For velocities below compressible flow limits, the use of the conservative form of governing equations will give meaningful results that closely represent the physics of the problem, which has addition and depletion of mass in the control volumes at the evaporator and condenser section of the heat pipe, respectively. In such a formulation, the pressure gradient can be removed from the energy equation for the incompressible flow situation encountered in low or moderate velocities, and the system pressure buildup can be accounted for by using the variable density formulation of the momentum equations.[12,15] With this, the governing equations take the form shown as case 2, in Table 1.

Case 3: Constant Density Formulation

In the case of low-velocity flow in the heat pipe, the constant density Navier–Stokes equations can approximate the physical problem. Nondimensional forms can be derived for this situation, which simplifies the system of equations. The dimensionless equations are shown as case 3 in Table 1.

In systems such as thermosyphons or micro heat pipes, the liquid flow is not confined to a wick, and the mass addition or depletion produces a local change in the cross-sectional area of the fluid path. A derivation of the governing equations introducing a varying cross-sectional area and accounting for the distribution of the local meniscus radius of curvature, as discussed in the literature,[16,17] is required to yield the appropriate governing equations in such cases. For these cases also, nondimensional formulations can be developed utilizing the strategy just described.

Conclusions

A procedure for deriving dimensionless governing equations for the analysis of the fluid flow and heat-transfer processes in heat pipes has been described in this technical Note. The characteristic dimensions chosen for the velocities are based on appropriate limiting Reynolds-number values for the region analyzed. Dimensionless equations are presented for the cases of compressible and incompressible flow situations applicable to heat pipes. It is expected that the use of these dimensionless equations in the computational analysis of heat pipes would be helpful in developing generalized numerical procedures for fluid flow and heat-transfer calculations and performance prediction in heat pipes, irrespective of their physical dimensions.

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